Provision Point Reverse Auction: A New Auction Mechanism with Applications for Conservation Contracts

Steven Otto*

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Abstract

Rent-seeking behavior by participants in payment for environmental services auctions reduces the number of affordable contracts and decreases environmental protection. I propose a new auction mechanism, the provision point reverse auction (PPRA), to mitigate this rent-seeking behavior. The PPRA includes a public component where the probability of contract acceptance for one individual is affected by the offers of others. I provide theoretical support for the new mechanism which proves that optimal offering behavior in a PPRA results in less rent-seeking from sellers than a multiunit reverse discriminative auction, even in contexts where participants are risk neutral and place no utility on the welfare of their peers. I follow this theoretical work with laboratory experiments comparing the PPRA to the multiunit reverse discriminative auction and the reverse budget-constrained auction. The experiments yield average offers between 12.57% to 58.17% smaller in a PPRA compared to the alternate reverse discriminative auctions, with the exact value dependent upon the compared mechanism and the target number of contracts. The experimental results are also compared to the theoretical predictions for a uniform price auction. I find that the PPRA is less expensive than the uniform price auction, while the effect on social efficiency is dependent on parameter values. If the goods being purchased are associated with positive externalities, as we would expect in PES or conservation contexts, the reduction in rent-seeking behavior can increase total surplus.

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*sgo8@cornell.edu. 363 Warren Hall, Cornell University, Ithaca, NY 14850
1 Introduction

Payment for environmental services (PES) programs have become an increasingly important component of conservation and environmental protection. Many of these programs use reverse (or procurement) discriminative auctions to allocate contracts to individuals who provide the environmental service [Latacz-Lohmann & Schilizzi, 2005]. Reverse discriminative auctions involve one buyer and many sellers, where the winners of the auctions receive their offer (or bid) as payment. In most reverse discriminative auctions, the buyer has a fixed budget and accepts offers in ascending order until the budget has been exhausted. In such an auction, sellers must balance potential gains in expected profit from a higher offer against corresponding decreases in the probability the offer will be given a contract by the buyer. The higher the offer, the less likely a contract will be won and the potential profit will be realized. If these auctions are conducted for multiple rounds, sellers gain information about the costs of their peers each round and use that information to increase their profits at the expense of the buyer. More specifically, sellers slowly increase their offers until they discover the offer at which they would no longer receive a contract. I call submitting an offer above one’s value “rent-seeking offers” or simply rent-seeking behavior. Over time, as rent-seeking behavior becomes more pronounced, the buyer can afford fewer contracts and incurs a welfare loss. This is a particularly significant problem for payment for environmental services or conservation programs because each contract may provide an environmental positive externality. In such a case, a reduction in the number of contracts the buyer can purchase could decrease the environmental benefits from the program and harm society at large. Given the large number of PES or conservation programs which use reverse discriminative auctions, including the Conservation Reserve Program (CRP) in the United States, the Auction for Landscape Recovery (ALR) in Australia, Challenge Funding in Scotland, and others, rent-seeking behavior is likely decreasing social welfare substantially.

With an eye toward mitigating rent-seeking behavior, and thus potentially increasing

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¹Much of the literature uses the term “bid-shading” instead of rent-seeking offers. This term is not appropriate for reverse auctions, however, as “bid-shading” literally means to make a slight reduction in a bid, while in reverse auctions, individuals seek to increase profits by increasing their offers.
social welfare in conservation and PES contexts, I designed the “provision point reverse auction” (PPRA). The PPRA functions as a discriminative reverse auction in that there is one buyer with many sellers and each individual with an accepted offer receives their offer as payment. However, unlike other discriminative procurement auctions, in a PPRA the buyer declares a requirement that a prespecified number of offers must be affordable for any offers to be accepted. That is, if the buyer cannot afford to purchase that prespecified number of offers, given their budget constraint, then no contracts will be made with any individual.

Similar to a reverse discriminative auction, an individual participating in a provision point reverse auction must weigh increases in potential profit from a higher offer against corresponding decreases in the probability of realizing that profit. As an individual’s offer becomes larger, it is also larger relative to the offers of their peers which decreases the probability the offer will be given a contract by the buyer. In a PPRA, however, a higher offer not only increases the offer relative to its peers, it also reduces the chance that the buyer can afford the prespecified number of units, which further lowers the probability of contract acceptance. I prove that this additional requirement incentivizes participants in a PPRA to submit offers closer to their costs, relative to a multiunit reverse discriminative auction.

The PPRA also includes a public component which serves as additional motivation for the mechanism. When an individual increases their offer, they negatively affect the expected profit of the other individuals in the auction by reducing the chance that any contracts are provided by the buyer. Thus, if individuals in a PPRA place positive utility on higher profits for their peers, they will be further incentivized to keep their offers close to their true costs. The author believes this is particularly likely to be true in close-knit rural or developing communities where PES programs are often implemented.

The PPRA would also make an attractive choice to governments or NGOs when the organizations are faced with thresholds for environmental value. For example, suppose a government agency is interested in restoring a polluted lake and reintroducing several species of fish. The agency estimates that a pollution reduction of X% would be required for the
water to be habitable for the fish. If the agency was to use a budget-constrained auction to pay neighboring individuals to abate emissions, the agency would have no guarantee that the contracts necessary for the reintroduction of the fish would be affordable. Since the primary goal of the program is to reintroduce fish to the lake, the agency would be wasting their money if the pollution abatement totaled less than $X\%$ of total pollution. If the agency instead used a PPRA to pay individuals to reduce their emissions, the agency would either achieve the pollution reduction necessary for the reintroduction of the fish or keep their budget and attempt some other PES program. This hypothetical situation is similar to a voluntary agreement between local New York farmers and New York City over the Catskill-Delaware water system [Appleton 2002]. Instead of paying the farmers to implement environmentally friendly practices, the city threatened that if 15% of the farmers did not participate, costly regulation would take effect to achieve the desired water quality improvement.

This paper provides theoretical evidence which shows that, under various assumptions, optimal offers under a PPRA are less than the optimal offers under a multiunit reverse discriminative auction, given an opportunity cost. These theoretical predictions are supported by evidence from laboratory experiments. The experimental work presented here abstracts away from the public component of the mechanism to focus on proof of concept. Ten experimental sessions were conducted with 240 student participants in total. The experimental results suggest that the PPRA reduces accepted offers by between 21.55\% to 58.17\% or 12.57\% to 21.59\%, when compared to a multiunit reverse discriminative auction or a budget-constrained multiunit reverse discriminative auction, respectively, with the exact value dependent upon the target number of contracts. The effect on offering behavior is particularly pronounced for the lowest offers, which are also the offers of greatest interest to the buyer.

2 Literature Review

Environmental goods or services are generally not exchanged on open markets and so do not have an easily observable price. Auctions are a convenient method for exchange when
the values for a good are unknown, and thus present an attractive choice to policy makers interested in purchasing environmental services. However, there are many types of auctions and it is not a priori obvious which auction format should be chosen, if an auction should be used at all. Vickrey’s seminal papers on auction theory, and much of the literature that followed, relies upon a “single independent private values” (SIPV) model (Vickrey, 1961, 1962). In the SIPV model: 1) there is a single indivisible unit available for sale, 2) each bidder knows their own private valuation, 3) all bidders are identical, 4) the valuations are independent and identically distributed, and 5) all bidders are risk neutral (Wolfstetter, 1996). Within this framework, there are four formats: the Dutch auction, the English auction, the first-price sealed bid auction, and the second-price sealed bid auction. The famous Revenue Equivalence Theorem (RET) states that the auctioneer receives the same revenue, regardless of the chosen format (Myerson, 1981; Riley & Samuelson, 1981; Vickrey, 1961). However, markets for environmental services do not satisfy many of the assumptions required for the RET to hold, and so we cannot apply this useful result to questions regarding conservation and PES auctions.

One key difference between conservation and PES auctions and auctions in an SIPV model is that auctions for environmental services are generally multiunit procurement auctions. That is, conservation or PES auctions generally involve one buyer purchasing multiple units of a good from multiple sellers. Unfortunately, the literature is less developed on the topic of multiunit procurement auctions than on other mechanisms, particularly for auctions where the buyer is restricted by a budget (Nautz, 1995; Bower & Bunn, 2001; Hailu, Schilizzi, & Thoyer, 2005; Latacz-Lohmann & Schilizzi, 2005; Schilizzi & Latacz-Lohmann, 2007). Harris and Raviv (1981) and Cox et al. (1984) generalized Vickrey’s original results and provided optimal offer functions for multiunit discriminative auctions with symmetric, risk neutral sellers whose costs are drawn from a uniform distribution. Hailu et al. (2005) extended this research and provided the optimal offer function for the reverse (or procurement) multiunit discriminative auction, which they call a “target-constrained” (as opposed to budget-constrained) auction. To the best of my knowledge, no one has specified an op-
timal offer function for a multiunit procurement auction where the buyer is constrained by a budget. Without more robust theoretical guidance from the literature, researchers and policy makers are forced to rely on experience and experimental evidence when making their decisions about how to purchase environmental services.

When using auctions in payment for environmental services (PES) programs, buyers frequently opt for a uniform second price or discriminative auction (Latacz-Lohmann & Schilizzi, 2005). In a uniform second price procurement auction, all individuals who submit winning offers are paid the first rejected offer. In settings where each seller has only one unit of the good to sell, individuals have the incentive to offer their true cost to the seller because increasing one’s offer cannot increase their own payoff. In a discriminative procurement auction, individuals who submit winning offers receive their offers as payment, analogous to a first price auction. Unlike the uniform second price procurement auction, in the discriminative procurement auction the optimal offering strategy is to submit an offer higher than one’s true cost. Because only individual sellers have full information on their true costs, this offering behavior leads to information rents for the sellers.

There is disagreement in the literature about the relative cost effectiveness of the uniform second price and discriminative auctions from the perspective of the buyer (Cason & Gangadharan, 2004; Goswami, Noe, & Rebello, 1996; Boxall, Packman, Weber, Samrawickrema, & Yang, 2009). Each mechanism’s efficiency and cost effectiveness is a function of the cost structure of the individual participants and the assumptions regarding information and communication. In their comprehensive review on the theoretical and empirical literature regarding conservation contracts, Latacz-Lohmann and Schilizzi (2005) provided several reasons that explain why funding agencies often choose discriminative procurement auctions over uniform price auctions, including the different risks of each auction mechanism for the buyer, which sellers profit the most from which auction, and the complexity of the different mechanisms.

To increase the efficiency of PES or conservation programs which use discriminative auction formats, I propose the provision point reverse auction (PPRA). The PPRA functions
as a discriminative procurement auction with the added requirement that a certain number of units are purchased by the buyer, given a constant, exogenous budget. For example, if the provision point requirement is 80% participation, but the buyer can only afford contracts for 75% of the sellers, then no contracts will be offered and the buyer will keep their money. Section 3 will provide further details.

The PPRA is connected to the research conducted on the provision point mechanism (PPM) for voluntary contributions to public goods (Davis & Holt, 1993; Marks & Croson, 1998; Rondeau, Schulze, & Poe, 1999; Rondeau, Poe, & Schulze, 2005). In a provision point mechanism, a public good is provided only if the total contributions exceed some predetermined threshold. If the total contributions do not exceed this “provision point,” then all contributions are refunded to the participants and no amount of the public good is provided. The PPRA is essentially the reverse auction form of the provision point mechanism: instead of a total contribution requirement, the sum of the lowest cost offers must be less than the budget and the potential profits from the auction can be viewed as the public good offered through the mechanism.

The closest paper to the provision point reverse auction, as formulated here, is Bush et al. (2013), who attempted to reduce the upward bias in willingness to accept estimates in a contingent valuation study using a provision point. Their mechanism is called a provision point mechanism (PPM), after the previous literature on contributions to public goods (Bush et al., 2013). This paper expands upon Bush et al. (2013) by generalizing their mechanism to an auction with many possible provision point requirements and tests the auction mechanism with real money in an experimental setting. This paper additionally provides theoretical support to substantiate the experimental evidence.

Much of the recent literature on conservation auctions has focused on special features of the conservation and PES settings which create complications when deciding which auction mechanism is most appropriate. Examples include environmental benefits as a function of the spatial location of the conserved land (Parkhurst et al., 2002), the availability of public information on historical auction results (Messer et al., 2017), and contract compliance as a
function of mechanism choice (Jack, 2013). This paper abstracts away from these issues to focus on proof of concept for the mechanism. Future research will examine how the PPRA interacts with some of these factors.

3 Theory and Model

The theory section is split into two parts. In the first subsection, I re-derive the symmetric Bayesian Nash Equilibrium optimal offer function for a multiunit reverse discriminative auction and demonstrate several properties of that optimal offer function. The second subsection introduces the provision point reverse auction, characterizes its expected profit function, and derives predictions for optimal behavior in a PPRA compared to a multiunit reverse discriminative auction.

3.1 Multiunit Reverse Discriminative Optimal Offer Function

Let $n \in \mathbb{N}$ denote the number of participants in an auction. In a multiunit reverse discriminative auction, the buyer is interested in purchasing $p \in \mathbb{N}$ units of a good from the $n$ sellers. This paper refers to $p$ as the “target” of the auction. Further, let $B \in (0, \infty)$ denote the budget if the auction is a budget-constrained multiunit auction, $v_i \in [0, 1]$ denote individual $i$’s opportunity cost or value, $o_i \in [0, \infty)$ denote their offer, $O_j(v_j)$ denote the assumed offering behavior of the other participants as a function of their values, and $O_j^{-1}(o_j)$ denote its inverse. To simplify the theory and computations, this paper makes the common assumption that all values are drawn from a standard uniform distribution. All of the auctions considered have the following properties:

1) More than one unit is being exchanged in each round;

2) The auctions have one seller with multiple buyers. These auctions are known as reverse (or procurement) auctions;

3) Values (opportunity costs) are independently drawn, so an individual’s value provides no information about the values of the other participants;
4) Each bidder knows their own value but they do not know the value of any other participant;

5) All participants, as well as the units they are trying to sell, are symmetrical and indistinguishable;

In addition, this paper also assumes all participants are risk neutral.

Much of the following theory relies upon order statistics, so a brief set of definitions is in order. (See Wolfstetter (1996) for a brief and exceedingly useful overview of order statistics and auction theory.) Out of a set of \( n \) draws from a distribution with probability density function \( f(x) \) and cumulative distribution function \( F(x) \), the random variable \( V_{(r)} \), which represents the \( r^{th} \) lowest draw, is called the \( r^{th} \) order statistic. The probability density function of \( V_{(r)} \) is given by

\[
f_{V_{(r)}}(x) = \frac{n!}{(r-1)!(n-1)!} F(x)^{r-1} (1 - F(x))^{n-r} f(x)
\]

(1)

For a standard uniform distribution, \( f(x) = 1 \) and \( F(x) = x \), so that the above simplifies to

\[
f_{V_{(r)}}(x) = \frac{n!}{(r-1)!(n-1)!} x^{r-1} (1 - x)^{n-r}
\]

(2)

Notice that this is a beta distribution, \( B(r, n+1-r) \).

The auction formats considered have expected profit functions given by:

\[
E[\Pi] = (o_i - v_i) \times Pr(o_i \in O)
\]

(3)

where \( O \) is the set of accepted contracts. The form of \( Pr(o_i \in O) \) depends on the auction used, as well as the parameter values chosen. As an individual increases their offer, potential profit, given by \((o_i - v_i)\), increases, but \( Pr(o_i \in O) \), the probability of realizing the potential profit, decreases. Thus, picking the optimal offer for a given value requires balancing these two effects.
For the multiunit reverse discriminative auction auction, expected profit is given by:

\[ E[\Pi] = (o_i - v_i) \times Pr(o_i < O_{(p)}) \]  

(4)

where \( O_{(p)} \) is the \( p^{th} \) lowest offer submitted by the other participants. From [2] above and assuming that all values are drawn from a standard uniform distribution, the probability that an individual’s offer is one of the \( p \) smallest offers is given by the function:

\[ g(n, p, O_{(p)}^{-1}(o_i)) = \frac{(n - 1)!}{(p - 1)!(n - p - 1)!} \int_{O_{(p)}^{-1}(o_i)}^{1} u^{p-1}(1 - u)^{n-p-1}du \]  

(5)

Intuitively, the \( g \) function takes in an individual’s offer, \( o_i \), and transforms it into an opportunity cost through \( O_{(p)}^{-1}(\cdot) \). \( O_{(p)}^{-1}(o_i) \) denotes the opportunity cost draw that would result in the offer \( o_i \) from the other participants in the auction, assuming the common offering behavior \( O_j(\cdot) \). This opportunity cost can then be used to calculate the probability the offer is one of the \( p \) smallest offers using the properties of order statistics and the given distribution for opportunity costs. From this point on, \( g(n, p, O_{(p)}^{-1}(o_i)) \) will be simplified as \( g(O_{(p)}^{-1}(o_i)) \).

Given an expected profit function, we are interested in the offer which, for each possible value, maximizes expected profit. That is, we are interested in a function which takes in an individual’s opportunity cost and returns their expected profit maximizing offer. Even more, we are interested in the offer function which is also a symmetric Bayesian Nash equilibrium. A symmetric Bayesian Nash equilibrium occurs when the best response to a given offer function is that offer function. More specifically, a symmetric Bayesian Nash equilibrium is an optimal offer function where, if an individual is participating in an auction where they assume the other individuals submit offers according to an offer function \( O_j(v_j) \), the optimal response is to also submit offers according to \( O_j(v_j) \).

Hailu, Schilizzi and Thoyer (2005) derive the symmetric Bayesian Nash equilibrium for a multiunit reverse auction. A rederivation and confirmation of their results is included in the
appendix. (Propositions 1 and 2 are expansions upon Hailu et al’s results.) In a multiunit reverse auction (also known as a target-constrained auction), a participant in the auction is interested in the probability that their offer will be one of the \( p \) lowest offers out of the \( n \) offers submitted by the \( n \) participants. The symmetric Bayesian Nash equilibrium for a multiunit procurement auction is:

\[
O^*_{i,TC}(v_i) = \frac{\int_{v_i}^{1} u^p(1-u)^{n-p-1}du}{\int_{v_i}^{1} u^{p-1}(1-u)^{n-p-1}du}
\]  

(6)

The optimal offer function, \( O^*_{i,TC}(v_i) \), takes in an individual’s value and returns the optimal offer (i.e., the offer which maximizes expected profit) for that value. Figure 1 displays this optimal offer function, assuming \( n = 8 \) and \( p = 5 \) or \( p = 3 \), where it can be clearly seen that low-value individuals can extract substantial rents (equivalent to many times their opportunity costs) from the buyer. Intuitively, for lower opportunity costs, an individual can increase their offer above their opportunity cost to increase potential profits while only slightly decreasing the probability that their offer will receive a contract. On the other hand, when a high opportunity cost individual submits an offer higher than their opportunity cost, they have a small chance that their offer will be accepted. As a result, the optimal offer function converges to cost revealing offers as an individual’s opportunity cost approaches 1.

Note that \( O^*_{i,TC}(v_i) \) is not defined when \( v_i = 1 \). Despite this, we can still make the following claim.

**Proposition 1:** As \( v_i \) approaches 1, \( O^*_{i,TC}(v_i) \) converges to 1.

**Proof.** For all \( v_i \in (0, 1) \), the numerator of \( O^*_{i,TC}(v_i) \) is less than the denominator, so \( O^*_{i,TC}(v_i) \) is bounded above by 1 for \( v_i \in (0, 1) \). Further, given that a non-negative expected profit requires \( O^*_{i,TC}(v_i) \geq v_i \), \( O^*_{i,TC}(v_i) \) is bounded below by \( v_i \). Both \( y = v_i \) and \( y = 1 \) converge to 1 as \( v_i \) approaches 1, so \( O^*_{i,TC}(v_i) \) converges to 1 as \( v_i \) approaches 1 by the sandwich theorem.  

It is also informative (and will be useful in future proofs) to show that \( O^*_{i,TC}(v_i) \) is a
strictly increasing function in $v_i$. But first, the following proposition and proof are made simpler by rewriting $O^*_{i,TC}(v_i)$ with the regularized beta function, given by:

$$I_x(a, b) = \int_0^x t^{a-1}(1 - t)^{b-1}dt$$

To rewrite $O^*_{i,TC}(v_i)$ in terms of the regularized beta function, we multiply the numerator and denominator of $O^*_{i,TC}(v_i)$ by $\frac{B(p + 1, n - p)}{B(p + 1, n - p)}$, where $B(p + 1, n - p)$ is represents the beta function with parameters $p + 1$ and $n - p$. This yields

$$O^*_{i,TC}(v_i) = \frac{\int_{v_i}^1 w^p(1 - u)^{n-p-1}du \times \frac{B(p + 1, n - p)}{B(p + 1, n - p)}}{\int_{v_i}^1 w^{p-1}(1 - u)^{n-p-1}du \times \frac{B(p, n - p)}{B(p, n - p)}} = \frac{1 - I_{v_i}(p + 1, n - p)}{1 - I_{v_i}(p, n - p)} \times \frac{B(p + 1, n - p)}{B(p, n - p)}$$

(7)
Given that $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, (7) simplifies to

$$O^*_{i,TC}(v_i) = \frac{1 - I_{v_i}(p + 1, n - p)}{1 - I_{v_i}(p, n - p)} \times \frac{p}{n} \quad (8)$$

One convenient property of the regularized beta is that

$$I_{v_i}(p + 1, n - p) = I_{v_i}(p, n - p) - \frac{v_i^p(1 - v_i)^{n-p-1}}{pB(p, n - p)} \quad (9)$$

and thus (8) can be rewritten as:

$$O^*_{i,TC}(v_i) = \frac{1 - I_{v_i}(p, n - p) + \frac{v_i^p(1 - v_i)^{n-p-1}}{pB(p, n - p)}}{1 - I_{v_i}(p, n - p)} \times \frac{p}{n} = \frac{p}{n} + \frac{v_i^p(1 - v_i)^{n-p-1}}{nB(p, n - p)(1 - I_{v_i}(p, n - p))} \quad (10)$$

**Proposition 2:** $O^*_{i,TC}(v_i)$ is a strictly increasing function of $v_i$ for $v_i \in [0, 1)$.

See Appendix 9.2 for the proof for Proposition 2. This proposition will prove critical when comparing optimal offering behavior between the multiunit reverse discriminative auction and the provision point reverse auction.

### 3.2 Set-up for Provision Point Reverse Auction

The provision point reverse auction is a discriminative reverse auction with the added requirement that $p$ of the $n$ total offers must be affordable for any contracts to be made, given the exogenous budget $B$. I call this additional requirement the “provision point requirement.” In a PPRA, an individual has to consider several factors when choosing their offer. Like most discriminative auctions, the individual must weigh the increase in potential profit from a higher offer against the decreased probability that a given offer will be accepted. In a PPRA, a higher offer decreases the probability of contract acceptance through two avenues. First, a higher offer makes it less likely that the offer will be one of the $p$ lowest offers, and thus less likely that the offer will receive one of the $p$ possible contracts. Second, a higher offer decreases the probability that the provision point requirement will be met, and thus
reduces the probability that any contracts will be provided.

The provision point requirement can be viewed as an “average” reservation price. In reverse auctions, a reservation price is the highest acceptable offer a seller can make to the buyer. By setting the budget and the provision point, the buyer implies that they will not spend more than $B/p$, on average, for the $p$ units. The average reservation price allows individuals with opportunity costs higher than the average reservation price to receive a contract by incentivizing lower opportunity cost individuals to submit lower offers. For example, in a PPRA, an individual can submit a bid higher than $B/p$ and still receive a contract if at least one of the other $p$ lowest offers is less than $B/p$, while this is not possible in an auction with a reservation price of $B/p$.

Looking back to (3), in a PPRA, the probability that an offer, $o_i$, receives a contract is the probability that $o_i$ is one of the $p$ lowest offers times the probability that the provision point requirement is met given that $o_i$ is one of the $p$ lowest offers. If either the provision point requirement is not met or $o_i$ is not one of the $p$ lowest offers, then $o_i$ will not receive a contract. Thus, the expected profit function for an individual participating in a PPRA is given by

$$E[\Pi] = (o_i - v_i) \times Pr(o_i < O(p)) \times Pr\left(\sum_{j=1}^{p-1} O(j) + o_i \leq B|o_i < O(p)\right)$$

(11)

where the third term on the right-hand side is the probability that the provision point requirement is met, given that $o_i$ is one of the $p$ lowest offers.

When considering the probability the provision point requirement will be met, an individual is interested in the expected value of the excess budget, given the sum of the expected offers of the other low cost individuals. That is, the individual is interested in the difference between the budget and what they expect the sum of the other $p - 1$ lowest bids to be. If their offer is one of the $p$ lowest and is greater than the excess budget, the provision point requirement will not be met because the sum of the $p$ lowest offers will exceed the budget. On the other hand, if their offer is one of the $p$ lowest and is less than the excess budget, the provision point requirement will be met as the sum of the $p$ lowest offers will be less than
the budget. If we assume that the other individuals submit offers according to a common offer function, \( O_j(\cdot) \), and we assume the budget, \( B \), is given exogenously, then the expected value of the excess budget given that \( o_i \) is one of the \( p \) lowest offers, denoted by \( \Theta \), is

\[
E[\Theta] = B - \sum_{j=1}^{p-1} E[O_j(v(j))|o_i < o_{(p)}]
\]  

(12)

where \( v(j) \) is the jth lowest opportunity cost. Intuitively, the expected value of the excess budget tells an individual the expected offer which, on average, would just meet the provision point requirement. The variance in the distribution of the excess budget suggests the degree to which the probability the provision point requirement will be met changes with small changes in an individual’s offer. Gupta and Sobel (1958) show that the sum of standard uniform order statistics is asymptotically normal. Thus, if the assumed offering behavior, \( O_j(v_j) \), is cost-revealing, then this distribution would be asymptotically normal. However, because individuals will not submit cost-revealing offers, we cannot use this approximation. In fact, given that the offer function for other individuals will generally not have a closed form, it seems unlikely that a closed form representation of (12) exists.

To summarize, an individual’s probability of submitting one of the \( p \) lowest offers, given their offer and assumed offering behavior of other individuals, is described by (5). Given the individual submits one of the \( p \) lowest offers, the probability that the provision point requirement is met is given by the probability that \( o_i \) is less than the excess budget, with the expected value of the excess budget given in (12).

Before we proceed further, we require the following axiom which is suggested by Proposition 1.

**Axiom 1:** If the probability that an individual receives a contract is 0 in any auction, then their optimal offering behavior is to submit an offer at their opportunity cost.

This axiom is important because it defines optimal offering behavior for values for which the optimal offer function might not be defined. For example, the optimal offer function for
the multiunit reverse discriminative auction (see (6)) is not defined when \( v_i = 1 \). A natural conclusion from this fact is that the optimal offer for individuals with \( v_i = 1 \) is 1 in both the multiunit reverse discriminative auction and the provision point reverse auction. With this background, I provide the following proposition.

**Proposition 3:** Suppose \( O_{i,TC}^*(v_i|n,p) \) is the symmetric Bayesian Nash equilibrium optimal offer function for the multiunit reverse discriminative auction with a target of \( p < n \) and \( O_{i,PP}^*(v_i|n,p,B) \) is a symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of \( p < n \) and a budget of \( B \). (From this point on, \( O_{i,TC}^*(v_i|n,p) \) and \( O_{i,PP}^*(v_i|n,p,B) \) will be simplified as \( O_{i,TC}^*(v_i) \) and \( O_{i,PP}^*(v_i) \), respectively.) Additionally, assume Axiom 1 holds. Then \( O_{i,TC}^*(v_i) = O_{i,PP}^*(v_i) \) if and only if either i) any single participant in the auction cannot affect the probability that the provision point requirement is met by increasing or decreasing their offer or ii) \( v_i = 1 \).

See Appendix 9.3 for the proof for Proposition 3. This proposition provides our first theoretical prediction: when the parameters of a PPRA are such that no single participant can affect the probability that the provision point requirement is met, the optimal offer function for all participants in the auction is the optimal offer function for a multiunit reverse discriminative auction. Proposition 4 expands upon Proposition 3.

**Proposition 4:** Suppose \( O_{i,TC}^*(v_i) \) is the symmetric Bayesian Nash equilibrium optimal offer function for the multiunit reverse discriminative auction with a target of \( p < n \) and \( O_{i,PP}^*(v_i) \) is a symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of \( p < n \) and a budget of \( B \). Further suppose that \( O_{i,TC}^*(v_i) \) is convex in \( v_i \).\(^2\) If a participant in the auction can impact the probability that the provision point requirement is met, then \( O_{i,TC}^*(v_i) > O_{i,PP}^*(v_i) \) for all \( v_i \).

\(^2\)The convexity assumption holds for every set of parameter values I have tested.
See Appendix 9.4 for the proof for Proposition 4. Propositions 3 and 4 tell us that we expect the optimal offer curve for the PPRA to be weakly below the optimal offer curve for a multiunit reverse discriminative auction with the same parameter values. The degree to which the optimal offer curve for the PPRA lies below the optimal offer curve for the multiunit reverse discriminative auction depends on the parameter values chosen. Note that these propositions do not make assumptions about the uniqueness of the symmetric Bayesian Nash equilibrium for the provision point reverse auction, but they do state that any symmetric Bayesian Nash equilibria for the PPRA must be less than the symmetric Bayesian Nash equilibrium for the target-constrained auction.

4 Experimental Design and Protocol

To test the theoretical predictions, ten experimental sessions were conducted with a total of 240 undergraduate student participants in Cornell University’s Lab for Experimental Economics and Decision Research (LEEDR). Informed consent was obtained from all participants, in accordance with IRB regulations. The ten sessions were divided into five treatments of two sessions each. The five treatments consisted of one “budget-constrained” multiunit reverse discriminative treatment with a budget of $4.42, two “target-constrained” multiunit reverse discriminative treatments with targets of five and three, and two provision point treatments with a budget of $4.42, one with a provision point requirement of five and the other with a provision point requirement of three. For clarity and to ease comparisons between treatments, this paper will, from this point on, refer to multiunit reverse discriminative auctions as “target-constrained” auctions and multiunit reverse discriminative auctions with budgets as “budget-constrained” auctions. Each session lasted at most 40 minutes. Student participants were not allowed to participate in more than one session. Average earnings were approximately $24 for each participant, with a range from $12 to $35. In each session, the 24 students were split evenly into three groups. Before the start of each session, the participants were given written instructions, which are included in the appendix. These written instructions include the following information:
1. The number of participants in a group (8).

2. The target or provision point requirement (5 or 3), if relevant.

3. The budget ($4.42), if relevant.

4. The common distribution from which all opportunity costs were drawn, $U(0,2)$.\(^3\)

A group size of 8 was chosen because relatively small group sizes increased both the sample and the impact of the provision point requirement. For the first PPRA treatment a provision point requirement of $p = 5$ was chosen so that a relatively large number of participants could contribute to the provision point requirement, while a target of 6 or 7 individuals might have led to larger offers in the target-constrained auction. In addition, initial parameters were chosen so that the participants in the auction could not divide the budget equally among themselves. That is, the budget was selected so that the fifth highest opportunity cost in each group was larger than the budget divided by 5. If at least one of the five lowest opportunity costs is greater than the budget divided by the provision point requirement, we say the auction is “psychologically binding.” To test the robustness of the mechanism, these sessions were followed with an additional PPRA treatment but with a provision point requirement of 3 rather than 5. This second PPRA treatment was not psychologically binding, as the budget divided by 3 was larger than the third highest opportunity cost in all groups.

For the purpose of common knowledge, the author read from a series of PowerPoint slides which included an overview of the experimental instructions. After the PowerPoint presentation, all subjects participated in 5 practice rounds where parameter values varied. In each round, the participants selected an offer between $0 and $7, where $7 was set as the maximum allowable offer. After each round, the participants were informed whether their offer was accepted and how much they were paid. If they were in the provision point reverse auction treatment, they were also informed if the provision point requirement was

\(^3\)Note that individuals drew offers from a $U(0,2)$ distribution rather than a $U(0,1)$, as we assumed in the theory section. This decision was made after conducting a pilot experiment where individuals drew costs up to $1$. I found that, with such low opportunity costs, the individual rounds were not salient to the participants. Indeed, the participants became increasingly impatient as the session continued. As a result, I reduced the number of rounds to 16 and increased the maximum opportunity cost to $2$. 
met. After the five practice rounds, opportunity costs were re-randomized and a series of 8 rounds began where the budget, target, provision point requirement and opportunity costs for each individual were fixed. Before the 9th round, the groups and opportunity costs were randomized once more and another 8 rounds were conducted to end the experiment. The results of the experiments are described in Section 5.

5 Results

5.1 Difference in Means

The first comparison between auction formats is a simple unconditional difference in means test between treatments and within rounds. An unconditional difference in means test is appropriate because the opportunity costs for the participants were randomized before the experimental sessions and were identical across treatments. The experimental format provides two sets of 8 rounds which are pooled to increase statistical power. That is, the offers from Rounds 1 and 9 are considered jointly, the offers from Rounds 2 and 10 are considered jointly, and so on. Given the varying parameter values, average offers were compared between formats with comparable restrictions. That is, the target-constrained auction with a target of 5, the budget-constrained auction with a budget of $4.42, and the PPRA with a provision point requirement of 5 and a budget of $4.42 were compared, and then the target-constrained auction with a target of 3, the budget-constrained auction with a budget = $4.42, and the PPRA with a provision point requirement of 3 and a budget of $4.42 were compared. The results are given in Table 1 and 2 below.

In each table, columns (1), (2) and (3) provide the mean offers for each treatment in each set of rounds, while columns (4) and (5) provide the difference in means between the treatments and the PPRA. There are several important results in Table 1. First, the target-constrained treatment has higher average offers than either of the other two treatments. Indeed, the difference in means between the the provision point reverse auction and the target-constrained treatment is above $1 in most rounds. The theory predicted that average
Table 1: Mean Offers – Target = 5, Budget = $4.42, PPR = 5

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Mean Offers</th>
<th>Difference: PPRA &amp; (TC) (BC)</th>
<th>TC</th>
<th>BC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(PPRA) (1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>1 &amp; 9</td>
<td>1.124</td>
<td>2.716</td>
<td>1.383</td>
<td>1.593***</td>
</tr>
<tr>
<td></td>
<td>(0.450) (0.105)</td>
<td>(0.687)</td>
<td>(0.123)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>2 &amp; 10</td>
<td>1.115</td>
<td>2.440</td>
<td>1.401</td>
<td>1.324***</td>
</tr>
<tr>
<td></td>
<td>(0.487) (0.740)</td>
<td>(0.670)</td>
<td>(0.090)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>3 &amp; 11</td>
<td>1.143</td>
<td>2.389</td>
<td>1.372</td>
<td>1.246***</td>
</tr>
<tr>
<td></td>
<td>(0.498) (0.644)</td>
<td>(0.620)</td>
<td>(0.083)</td>
<td>(0.081)</td>
</tr>
<tr>
<td>4 &amp; 12</td>
<td>1.145</td>
<td>2.257</td>
<td>1.420</td>
<td>1.111***</td>
</tr>
<tr>
<td></td>
<td>(0.490) (0.501)</td>
<td>(0.859)</td>
<td>(0.071)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>5 &amp; 13</td>
<td>1.137</td>
<td>2.223</td>
<td>1.347</td>
<td>1.086***</td>
</tr>
<tr>
<td></td>
<td>(0.464) (0.430)</td>
<td>(0.529)</td>
<td>(0.065)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>6 &amp; 14</td>
<td>1.200</td>
<td>2.161</td>
<td>1.335</td>
<td>0.961***</td>
</tr>
<tr>
<td></td>
<td>(0.577) (0.371)</td>
<td>(0.508)</td>
<td>(0.070)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>7 &amp; 15</td>
<td>1.167</td>
<td>2.090</td>
<td>1.360</td>
<td>0.923***</td>
</tr>
<tr>
<td></td>
<td>(0.475) (0.328)</td>
<td>(0.550)</td>
<td>(0.059)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>8 &amp; 16</td>
<td>1.186</td>
<td>2.101</td>
<td>1.364</td>
<td>0.915***</td>
</tr>
<tr>
<td></td>
<td>(0.576) (0.604)</td>
<td>(0.587)</td>
<td>(0.085)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>All</td>
<td>1.152</td>
<td>2.297</td>
<td>1.373</td>
<td>1.115***</td>
</tr>
<tr>
<td></td>
<td>(0.507) (0.662)</td>
<td>(0.633)</td>
<td>(0.030)</td>
<td>(0.029)</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Note: The above table contains the mean for each of the three auction treatments and difference in means between the TC and BC auction treatments and the PPRA, with the standard deviations and standard errors below for the means or differences in means, respectively. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 5, a budget-constrained auction with a budget of $4.42 and a provision point auction with a provision point requirement of 5 and a budget of $4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.

offers would be higher in the target-constrained auction than the provision point reverse auction, but the magnitude of the differences was unexpected. Second, the budget-constrained treatment has higher average offers than the PPRA as well, albeit to a lesser extent. In most rounds, the budget-constrained treatment has offers more than $0.20 higher than its provision point counterpart. Third, notice that while the average offers are relatively stable across rounds for the PPRA and budget-constrained auction, the target-constrained auction saw
Table 2: Mean Offers – Target = 3, Budget = $4.42, PPR = 3

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Mean Offers (PPRA)</th>
<th>Mean Offers (TC)</th>
<th>Mean Offers (BC)</th>
<th>Difference: PPRA &amp; (TC) (BC)</th>
<th>TC</th>
<th>BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 9</td>
<td>1.249 (0.628)</td>
<td>1.631 (0.556)</td>
<td>1.383 (0.687)</td>
<td>0.382*** (0.086) (0.095)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 10</td>
<td>1.197 (0.561)</td>
<td>1.454 (0.407)</td>
<td>1.401 (0.670)</td>
<td>0.257*** (0.071) (0.089)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 11</td>
<td>1.252 (0.639)</td>
<td>1.409 (0.381)</td>
<td>1.372 (0.620)</td>
<td>0.157** (0.076) (0.091)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; 12</td>
<td>1.269 (0.672)</td>
<td>1.400 (0.457)</td>
<td>1.420 (0.859)</td>
<td>0.131 (0.083) (0.111)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 &amp; 13</td>
<td>1.239 (0.636)</td>
<td>1.362 (0.481)</td>
<td>1.347 (0.529)</td>
<td>0.123 (0.081) (0.084)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 &amp; 14</td>
<td>1.225 (0.601)</td>
<td>1.361 (0.695)</td>
<td>1.335 (0.508)</td>
<td>0.137 (0.094) (0.080)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 &amp; 15</td>
<td>1.319 (0.766)</td>
<td>1.339 (0.584)</td>
<td>1.360 (0.550)</td>
<td>0.021 (0.098) (0.096)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 &amp; 16</td>
<td>1.323 (0.922)</td>
<td>1.412 (0.785)</td>
<td>1.364 (0.587)</td>
<td>0.089 (0.124) (0.112)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>1.259 (0.685)</td>
<td>1.421 (0.563)</td>
<td>1.373 (0.633)</td>
<td>0.162*** (0.032) (0.034)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Note: The above table contains the mean for each of the three auction treatments and difference in means between the TC and BC auction treatments and the PPRA, with the standard deviations and standard errors below for the means or differences in means, respectively. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 3, a budget-constrained auction with a budget of $4.42 and a provision point auction with a provision point requirement of 3 and a budget of $4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.

its average offers decrease over time. This runs contrary to previously established theoretical results, which suggest offers increase over time in a target-constrained auction. Instead, it seems individuals submitted high offers in the first round, and their offers decreased over time as the participants competed over contracts. This may be a result of the relatively small group size, as conversation (and therefore collusion) between individuals was not permitted during the experiment. The statistically significant differences in means support the
claim that the PPRA can reduce offers when compared to the target- or budget-constrained treatments.

Table 2 provides the results from additional experiments with different parameter values, where both the target-constraint and the provision point requirement were set to 3. First, note that average offers are always less in the PPRA than either the target- or budget-constrained auction, but that the differences are not statistically significant in most rounds. This agrees with our intuitive expectations, where a smaller target with a constant budget is less restrictive than a larger target with the same budget. Indeed, these results are generally consistent with the contention that, even when the provision point requirement is not more restrictive than the target or budget constraint, the provision point auction provides lower average offers. Also note that, with these parameter values, the target- and budget-constrained auctions provide more comparable average offers than seen in Table 1 where the target-constrained auction resulted in substantially higher offers.

Tables 1 and 2 provide differences in means across all offers. The buyer, however, is primarily interested in the lowest offers because those offers actually receive contracts and result in payments from the buyer. Thus, a comparison of means of the lowest offers between auction formats would provide more information about improvements in the buyer’s welfare than a comparison of all offers. The difference in means for the lowest five offers between the target-constrained treatment with a target of five, the budget-constrained treatment with a budget of $4.42, and the PPRA with a budget of $4.42 and provision point requirement of five are given in Table 3. Table 3 shows comparable differences to Table 1 and provides additional support that the PPRA may be an attractive alternative to the target- and budget-constrained auctions from the perspective of the buyer. Indeed, the mean of the five lowest offers in a PPRA was between 19.4% and 25.6% smaller in the tested provision point reverse auctions than the comparable budget-constrained auction, depending on the round. One advantage of comparing the lower offers is that large outliers are removed from the comparison.

The difference in means for the lowest three offers between the target-constrained treat-
<table>
<thead>
<tr>
<th>Rounds</th>
<th>Mean Offers</th>
<th>Difference: PPRA &amp; TX BC</th>
<th>Difference: PPRA &amp; TC BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 9</td>
<td>0.822</td>
<td>2.160</td>
<td>1.042</td>
</tr>
<tr>
<td></td>
<td>(0.316)</td>
<td>(0.765)</td>
<td>(0.394)</td>
</tr>
<tr>
<td>2 &amp; 10</td>
<td>0.814</td>
<td>2.069</td>
<td>1.093</td>
</tr>
<tr>
<td></td>
<td>(0.292)</td>
<td>(0.608)</td>
<td>(0.320)</td>
</tr>
<tr>
<td>3 &amp; 12</td>
<td>0.838</td>
<td>2.104</td>
<td>1.095</td>
</tr>
<tr>
<td></td>
<td>(0.323)</td>
<td>(0.495)</td>
<td>(0.307)</td>
</tr>
<tr>
<td>4 &amp; 12</td>
<td>0.848</td>
<td>2.027</td>
<td>1.107</td>
</tr>
<tr>
<td></td>
<td>(0.323)</td>
<td>(0.397)</td>
<td>(0.306)</td>
</tr>
<tr>
<td>5 &amp; 13</td>
<td>0.854</td>
<td>2.025</td>
<td>1.089</td>
</tr>
<tr>
<td></td>
<td>(0.299)</td>
<td>(0.352)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>6 &amp; 14</td>
<td>0.885</td>
<td>2.016</td>
<td>1.103</td>
</tr>
<tr>
<td></td>
<td>(0.307)</td>
<td>(0.333)</td>
<td>(0.223)</td>
</tr>
<tr>
<td>7 &amp; 15</td>
<td>0.886</td>
<td>1.962</td>
<td>1.099</td>
</tr>
<tr>
<td></td>
<td>(0.325)</td>
<td>(0.281)</td>
<td>(0.223)</td>
</tr>
<tr>
<td>8 &amp; 16</td>
<td>0.870</td>
<td>1.934</td>
<td>1.094</td>
</tr>
<tr>
<td></td>
<td>(0.310)</td>
<td>(0.298)</td>
<td>(0.224)</td>
</tr>
<tr>
<td>All</td>
<td>0.852</td>
<td>2.037</td>
<td>1.086</td>
</tr>
<tr>
<td></td>
<td>(0.311)</td>
<td>(0.471)</td>
<td>(0.285)</td>
</tr>
</tbody>
</table>

**Note:** The above table contains the mean of the lowest five offers for each of the three auction treatments and difference in means between the five lowest offers for the TC and BC auction treatments and the PPRA, with the standard deviations and standard errors below for the means or differences in means, respectively. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 5, a budget-constrained treatment with a budget of $4.42, and the PPRA with a budget of $4.42 and provision point requirement of three are given in Table 4. Table 4 shows statistically significant differences in means between the three auction formats in most rounds, and thus suggests that the PPRA can yield improvements in the buyer’s welfare for an additional set of parameter values. More specifically, the mean of the
three lowest offers in tested provision point auctions was between 8.9% and 15.7% smaller than the comparable mean in the budget-constrained auctions, depending on the rounds. Indeed, Table 4 provides more compelling evidence than Table 2 that the PPRA can lower offers, even when the provision point requirement isn’t “psychologically binding.”

### Table 4: Mean Lowest 3 Offers – Pooled Rounds

<table>
<thead>
<tr>
<th>Rounds</th>
<th>PPRA</th>
<th>Mean Offers</th>
<th>Difference: PPRA &amp;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>TC</td>
<td>BC</td>
<td>TC</td>
</tr>
<tr>
<td>1 &amp; 9</td>
<td>0.750</td>
<td>1.142</td>
<td>0.823</td>
<td>0.392***</td>
</tr>
<tr>
<td></td>
<td>(0.279)</td>
<td>(0.346)</td>
<td>(0.284)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>2 &amp; 10</td>
<td>0.777</td>
<td>1.094</td>
<td>0.922</td>
<td>0.317***</td>
</tr>
<tr>
<td></td>
<td>(0.268)</td>
<td>(0.257)</td>
<td>(0.253)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>3 &amp; 12</td>
<td>0.845</td>
<td>1.100</td>
<td>0.937</td>
<td>0.255***</td>
</tr>
<tr>
<td></td>
<td>(0.237)</td>
<td>(0.248)</td>
<td>(0.266)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>4 &amp; 12</td>
<td>0.827</td>
<td>1.081</td>
<td>0.918</td>
<td>0.254***</td>
</tr>
<tr>
<td></td>
<td>(0.241)</td>
<td>(0.224)</td>
<td>(0.269)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>5 &amp; 13</td>
<td>0.836</td>
<td>1.033</td>
<td>0.979</td>
<td>0.197***</td>
</tr>
<tr>
<td></td>
<td>(0.238)</td>
<td>(0.204)</td>
<td>(0.231)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>6 &amp; 14</td>
<td>0.864</td>
<td>1.010</td>
<td>0.995</td>
<td>0.176***</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.219)</td>
<td>(0.190)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>7 &amp; 15</td>
<td>0.872</td>
<td>0.993</td>
<td>1.002</td>
<td>0.121**</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.223)</td>
<td>(0.200)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>8 &amp; 16</td>
<td>0.849</td>
<td>0.992</td>
<td>0.998</td>
<td>0.143***</td>
</tr>
<tr>
<td></td>
<td>(0.206)</td>
<td>(0.200)</td>
<td>(0.207)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>All</td>
<td>0.828</td>
<td>1.056</td>
<td>0.947</td>
<td>0.228***</td>
</tr>
<tr>
<td></td>
<td>(0.236)</td>
<td>(0.247)</td>
<td>(0.244)</td>
<td>(0.020)</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1

Note: The above table contains the mean of the lowest three offers for each of the three auction treatments and difference in means between the three lowest offers for the TC and BC auction treatments and the PPRA, with the standard deviations and standard errors below for the means or differences in means, respectively. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 3, a budget-constrained auction with a budget of $4.42 and a provision point auction with a provision point requirement of 3 and a budget of $4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.
5.2 Offer Functions

Figures 2 and 3 below display the fitted offer functions and individual offers (grouped by similar parameter values) observed from experiments across all rounds, assuming an exponential specification for the offer functions. The exponential specification was chosen both because of its similarity to the optimal offer curve for the target-constrained auction (see Figure 1) and because it fits the data well, particularly compared to either a linear or quadratic specification.

![Figure 2: Comparison of Offer Functions, PPR = 5: All Rounds](image)

These figures show the degree to which individuals submitted offers above their opportunity costs across the different treatments and for the different parameter values. In some instances, individuals actually submitted offers below their opportunity costs, represented by the 45 degree line. In a provision point reverse auction, it is possible that this behavior
was altruistic: some individuals decreased their offers below their opportunity costs in the hope of satisfying the provision point requirement, and thus allowing some of their peers to receive contracts. Why some individuals in the budget-constrained auction chose to submit offers below their opportunity cost is unclear, although the behavior was largely limited to a few participants. Each offer function is surrounded by a shaded region, representing a 95% confidence interval. Given the large variance in offers within treatments, I suggest greater consideration of the difference in average offers than the difference in coefficients on fitted functions. The variance in offers is consistent with individuals struggling to determine optimal offering behavior which is hardly surprising given the computational difficulty of determining an optimal offer for any of the three auction formats.

Figures 4 and 5 provide the offers and fitted offer curves for the first and ninth rounds and
the eighth and sixteenth rounds, respectively, for the treatments with a target or provision point requirement of five and a budget of $4.42, while Figures 6 and 7 provide similar representations of the data for treatments with a target or provision point requirement of 3 and a budget of $4.42. The first and ninth rounds are the initial rounds after groups have been re-randomized, while the eighth and sixteenth rounds are the final rounds before either re-randomization or the conclusion of the experiment.

Figure 4:

![Comparison of Offer Functions, PPR = 5: First Rounds (1 & 9)](image)

5.3 Efficiency Analysis

This paper is interested not only in comparing the three auction treatments with each other, but also against the theoretical predictions for the uniform reverse auction. In a uniform reverse auction, the buyer sets a target and the winning individuals receive the first rejected
offer as payment, similar to a Vickrey second price auction. Theoretically, we expect individuals in a uniform procurement auction will submit their opportunity costs as their offers. To compare the auction formats this paper uses three criteria to measure their efficacy. The first measure is social efficiency, which is defined as follows:

$$\text{Social Efficiency} = \frac{\sum_i^p v_{(i)}}{\sum_i^p v_i} \times 100$$ (13)

where $v_{(i)}$ is the ith smallest opportunity cost in the auction. In other words, social efficiency is the minimum opportunity cost required to supply five contracts divided by the opportunity cost of the individuals who received contracts. From society’s perspective, welfare is maximized when the lowest opportunity cost individuals receive the available contracts. However, this result is not necessarily the case in instances with positive externalities like one might
expect from PES programs. Nonetheless, the measure informative.

The second measure is simply the total cost to the buyer of purchasing \( p \) contracts. This allows us to compare cost savings for the buyer across the different auction mechanisms, and thus the amount of money the buyer must spend, on average, for the \( p \) units.

Finally, this paper uses a “cost effectiveness” measure to further compare how costly the auctions are for the buyer. This measure is defined as follows:

\[
\text{Cost Effectiveness} = \frac{\text{Uniform Auction Cost} - \text{Other Auction Cost}}{\text{Uniform Auction Cost} - \text{Total Opportunity Cost}}
\]  

By definition, if the participants submitted offers equal to their opportunity costs, the cost efficiency measure would be 100\%, while the cost efficiency measure is 0\% for the uniform auction.
Tables 5 and 6 below provide the efficiency and cost effectiveness measures for the various auctions by their parameter values. The OC column provides the measures for a hypothetical discriminative auction where individuals submit their opportunity costs as offers. In such an auction, all of the welfare gains would be given to the buyer and the auction would be 100% socially efficient. As such, it serves as the ideal auction from the perspective of the buyer. There are two important problems to discuss before continuing to the efficiency measures. First, we cannot compare the budget-constrained auction to the other formats directly with these measures because the buyer, in the experiments, was never able to afford 5 contracts in the budget-constrained treatment. Thus, questions including, “how much did it cost the buyer to purchase five contracts” are nonsensical for budget-constrained auction. Second, the provision point auctions didn’t always result in contracts in the treatment with PPR.
Table 5: Efficiency Measures, Target/PPR = 5

<table>
<thead>
<tr>
<th></th>
<th>OC</th>
<th>Uniform</th>
<th>TC</th>
<th>PPRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Efficiency</td>
<td>100%</td>
<td>100%</td>
<td>71.46%</td>
<td>95.98%</td>
</tr>
<tr>
<td></td>
<td>(14.89%)</td>
<td>(9.40%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Total Cost of</td>
<td>$3.02</td>
<td>$6.64</td>
<td>$10.19</td>
<td>$4.07</td>
</tr>
<tr>
<td>Providing 5 Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Effectiveness</td>
<td>100%</td>
<td>0%</td>
<td>-97.96%</td>
<td>71.12%</td>
</tr>
<tr>
<td></td>
<td>(29.65%)</td>
<td>(12.03%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The above table contains efficiency measures for several different auction formats. Standard errors are enclosed in parentheses below their given estimates. The OC column contains the results from a theoretical discriminative auction where all individuals submit their opportunity costs as offers. The Uniform column contains the predicted results from a uniform price auction. The TC column contains the experimental results for the target-constrained auction and the PPRA column contains the experimental results for the provision point reverse auction in rounds where the provision point requirement was met.

Unsurprisingly, given the theoretical predictions, the target-constrained auction performs the worst by all three measures, regardless of the parameter values. Indeed, the target-constrained auction costs over twice as much, on average, as the provision point reverse auction and costs nearly 80% more than the predictions for the uniform auction as well, when $p = 5$. On the other hand, the provision point reverse auction was only slightly less socially efficient than the predictions for the uniform auction when $p = 5$, although the PPRA achieved lower social efficiency than the predictions for the uniform price auction when $p = 3$. In summary, the PPRA performs better than the uniform price auction from the perspective of the buyer, while it performs slightly worse than the uniform price auction by social efficiency. However, the difference in the social efficiency measure is not statistically significant for the session where the provision point requirement was equal to 5.

= 5, as the provision point requirement wasn’t met in approximately 33% of the rounds. (The PPR was met in every round for the treatment with $PPR = 3$.) As a result, it isn’t always sensible to compare the PPRA to the target-constrained and uniform price auctions. Instead, this paper presents only the efficiency measure for the PPRA when the provision point requirement was met. This alters the efficiency estimates slightly when the PPR = 5, but does not alter the analysis when the PPR = 3.
Table 6: Efficiency Measures, Target/PPR = 3

<table>
<thead>
<tr>
<th></th>
<th>OC</th>
<th>Uniform</th>
<th>TC</th>
<th>PPRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Efficiency</td>
<td>100%</td>
<td>100%</td>
<td>64.61%</td>
<td>76.8%</td>
</tr>
<tr>
<td></td>
<td>(24.65%)</td>
<td>(22.85%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Total Cost of</td>
<td>$1.06</td>
<td>$2.64</td>
<td>$3.17</td>
<td>$2.48</td>
</tr>
<tr>
<td>Providing 3 Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Effectiveness</td>
<td>100%</td>
<td>0%</td>
<td>-33.82%</td>
<td>9.66%</td>
</tr>
<tr>
<td></td>
<td>(34.21%)</td>
<td>(35.92%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The above table contains efficiency measures for several different auction formats. Standard errors are enclosed in parentheses below their given estimates. The OC column contains the results from a theoretical discriminative auction where all individuals submit their opportunity costs as offers. The Uniform column contains the predicted results from a uniform price auction. The TC column contains the experimental results for the target-constrained auction and the PPRA column contains the experimental results for the provision point reverse auction in rounds where the provision point requirement was met.

6 Discussion

Given the structure of the PPRA, I believe it will be particularly effective when three criteria hold true, as discussed in the introduction. First, if there is a threshold of interest the PPRA can ensure the government either some level of environmental service or the welfare they obtain from retaining their budget and expending it on an alternative PES program.

Second, I believe that the PPRA will be particularly effective for auctions with small numbers of participants who all operate in a given region. As the number of participants in a PPRA becomes smaller, the ability of any individual to affect the probability the provision point requirement is met increases, which increases the impact of the provision point requirement on offering behavior (See Propositions 3 and 4). Further, I believe that individuals who know each other will be more likely to take the welfare of the other participants into account. As such, a PPRA which takes place in a particular region may increase the salience of the provision point requirement even further.

Finally, the PPRA will be most effective at reducing offers when the cost of running an auction is low and when the buyer can move the program to a new location when the provision point requirement isn’t met. The buyer may forgo substantial welfare opportunities
if they cannot eventually provide contracts to some individuals, and thus the ability to move the auction to a new location at relatively little cost will decrease the chance the buyer is not able to purchase some environmental service.

As an example of a setting which satisfies these three criteria, consider the BirdReturn© program in California. In the BirdReturn© program, rice farmers in the Central Valley of California are paid by conservationists and aviphiles to flood their paddies to create small habitats for migratory birds. The number of rice farmers in a given area is relatively small, and if a certain number of these “pop-up habitats” are not created, then the birds will not be able to use the regions as stepping stones along their journey. There are several potential areas in the Central Valley that could serve as pop-up habitats, so the conservationists and aviphiles could move to a new location if they cannot afford a certain number of contracts.

While the provision point reverse auction has the potential to function well in some settings, it certainly would not be appropriate for all procurement auctions. For example, electricity markets use reverse auctions to allocate contracts to energy producers. A PPRA in this context would mean that no electricity would be produced when the provision point requirement is not met, which would be an unacceptable outcome given that demand for electricity is inelastic.

7 Conclusion

This paper introduces a new auction mechanism designed for conservation and PES settings. The Provision Point Reverse Auction has the potential to increase the efficiency and cost effectiveness of conservation and PES programs, while simultaneously decreasing uncertainty for the purchasers of the environmental goods. The experimental and theoretical results support this claim, showing that the PPRA can save the procurer between 21.55% to 58.17% or 12.57% to 21.59% of their costs on average, when comparing to a multiunit reverse discriminative auction or budget-constrained multiunit reverse discriminative auction, respectively, with the exact value dependent upon the target number of contracts. Further, the PPRA also improves social efficiency over the multiunit reverse discriminative auction,
reducing the total cost of the environmental service to society. Future research will expand the empirical support for the PPRA to field settings or continue theoretical examination to consider optimal offering behavior in multiple rounds. Settings where environmental benefits are a function of spatial proximity of conserved land provide a particularly fruitful area of research.

References

Appleton, A. F. (2002). How New York City used an ecosystem services strategy carried out through an urban-rural partnership to preserve the pristine quality of its drinking water and save billions of dollars. In The Katoomba Conference.


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8 Acknowledgements

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9 Appendix

9.1 Multiunit Reverse Discriminative Auction Symmetric Optimal Offer Function

I confirm the symmetric Bayseian Nash equilibrium found by Hailu, Schilizzi and Thoyer (2005). In a multiunit reverse auction (also known as a target-constrained auction), a partic-
Participant in the auction is interested in the probability that their offer will be one of the $p$ lowest offers out of the $n$ offers submitted by the $n$ participants. This probability is represented by $g(O^{-1}_j(o_i))$ in (5). The expected profit for an individual in this auction is then represented by

$$E[\Pi] = (o_i - v_i) \times g(O^{-1}_j(o_i))$$ (15)

which is a more specific representation of (4). The first order conditions to maximize (15) are

$$g(O^{-1}_j(o_i)) + (o_i - v_i) \frac{\partial g(O^{-1}_j(o_i))}{\partial o_i} \frac{\partial O^{-1}_j(o_i)}{\partial o_i} = 0$$ (16)

Recalling that

$$\frac{\partial f^{-1}(x)}{\partial x} = \frac{1}{f'(f^{-1}(x))}$$ (17)

Equation (16) simplifies to

$$g(O^{-1}_j(o_i)) + (o_i - v_i) \frac{\partial g(O^{-1}_j(o_i))}{\partial o_i} \frac{\partial O^{-1}_j(o_i)}{\partial o_i} = 0$$ (18)

In equilibrium, $o_i = O_j(v_i) = O_{i,TC}^*(v_i)$. Equation (18) becomes

$$v_i \frac{\partial g(v_i)}{\partial o_i} = g(v_i) \frac{\partial O_{i,TC}^*(v_i)}{\partial o_i} + O_{i,TC}^*(v_i) \frac{\partial g(v_i)}{\partial o_i}$$ (19)

Integrating both sides of (19) with respect to $o_i$ yields

$$- \int_{v_i}^{1} u \frac{\partial g(u)}{\partial o_i} du = O_{i,TC}^*(v_i)g(v_i)$$ (20)

Dividing both sides by $g(v_i)$ and noting that $g(1) = 0$, we have

$$O_{i,TC}^*(v_i) = \frac{- \int_{v_i}^{1} u \frac{\partial g(u)}{\partial o_i} du}{\int_{v_i}^{1} \frac{\partial g(u)}{\partial o_i} du}$$ (21)
Given that, according to (5),

\[
\frac{\partial g(u)}{\partial o_i} = \frac{(n - 1)!}{(p - 1)! (n - p - 1)!} u^{p-1} (1 - u)^{n-p-1}
\]  

the symmetric Bayesian Nash equilibrium for the multiunit reverse discriminative auction is given by

\[
O_{i,TC}^*(v_i) = \frac{\int_{v_i}^1 u^p (1 - u)^{n-p-1} du}{\int_{v_i}^1 u^{p-1} (1 - u)^{n-p-1} du}
\]

\[\text{(23)}\]

### 9.2 Proposition 2 Proof

**Proposition 5:** \(O_{i,TC}^*(v_i)\) is a strictly increasing function of \(v_i\) for \(v_i \in [0, 1)\).

**Proof.** Applying the quotient rule, the derivative of \(O_{i,TC}^*(v_i)\) with respect to \(v_i\) is

\[
\frac{\partial O_{i,TC}^*(v_i)}{\partial v_i} = \left[ \left( (pv_i^{p-1} (1 - v_i)^{n-p-1} + (n - p - 1)(1 - v_i)^{n-p-2} v_i^p \right) \times \right. \\
(nB(p, n - p)(1 - I_{v_i}(p, n - p))) + nv_i^p (1 - v_i)^{n-p-1} v_i^{p-1} (1 - v_i)^{n-p-1} \right] / \left. \right]
\]

\[
\left. n^2 B(p, n - p)^2 (1 - I_{v_i}(p, n - p))^2 \right)
\]

Factoring out \(v_i^{p-1}\) and \((1 - v_i)^{n-2p-2}\), and dividing the numerator and denominator by \(n\) yields

\[
\frac{\partial O_{i,TC}^*(v_i)}{\partial v_i} = \frac{v_i^{p-1} (1 - v_i)^{n-2p-2} ((1 - v_i)^p (p - nv_i + v_i) B(p, n - p)(1 - I_{v_i}(p, n - p)) + (1 - v_i)^n v_i^p)}{nB(p, n - p)^2 (1 - I_{v_i}(p, n - p))^2}
\]

\[\text{(25)}\]

We want to show that

\[
0 < \frac{v_i^{p-1} (1 - v_i)^{n-2p-2} ((1 - v_i)^p (p - nv_i + v_i) B(p, n - p)(1 - I_{v_i}(p, n - p)) + (1 - v_i)^n v_i^p)}{nB(p, n - p)^2 (1 - I_{v_i}(p, n - p))^2}
\]

\[\text{(26)}\]

for all \(v_i \in (0, 1)\). Note that \(v_i^{p-1} (1 - v_i)^{n-2p-2}\) and \(pB(p, n - p)^2 (1 - I_{v_i}(p, n - p))^2\) are both positive, and thus both can be cancelled out without affecting the direction of the inequality.
Inequality (26) thus holds when

\[- (1 - v_i)^p (p - n v_i + v_i) B(p, n - p) (1 - I_{v_i}(p, n - p)) < (1 - v_i)^n v_i^p \]  

(27)

Dividing both sides by \( n(1 - v_i)^p B(p, n - p) (1 - I_{v_i}(p, n - p)) \) yields

\[- \left( \frac{p}{n} - v_i + \frac{v_i}{n} \right) < \frac{(1 - v_i)^n v_i^p}{n B(p, n - p) (1 - I_{v_i}(p, n - p))} \]  

(28)

A slight rearrangement of terms yields

\[ v_i \left( 1 - \frac{1}{n} \right) < \frac{p}{n} + \frac{(1 - v_i)^n v_i^p}{n B(p, n - p) (1 - I_{v_i}(p, n - p))} \]  

(29)

Notice that the righthand side of (29) is the optimal offer function for \( O_{i,TC}^*(v_i) \) from (10).

Also note that \( (1 - \frac{1}{n}) < 1 \). Equation (29) thus implies (30) below.

\[ v_i \left( 1 - \frac{1}{n} \right) < v_i < O_{i,TC}^*(v_i) \]  

(30)

Given that profit maximization requires \( O_{i,TC}^*(v_i) > v_i \) for all \( v_i \in [0, 1) \), the optimal offer function is increasing for all \( v_i \in [0, 1) \).

\[ \square \]

### 9.3 Proposition 3 Proof

**Proposition 6:** Suppose \( O_{i,TC}^*(v_i|n, p) \) is the symmetric Bayesian Nash equilibrium optimal offer function for the multiunit reverse discriminative auction with a target of \( p < n \) and \( O_{i,PP}^*(v_i|n, p, B) \) is a symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of \( p < n \) and a budget of \( B \). (From this point on, \( O_{i,TC}^*(v_i|n, p) \) and \( O_{i,PP}^*(v_i|n, p, B) \) will be simplified as \( O_{i,TC}^*(v_i) \) and \( O_{i,PP}^*(v_i) \), respectively.) Additionally, assume Axiom 1 holds. Then \( O_{i,TC}^*(v_i) = O_{i,PP}^*(v_i) \) if and only if either i) any single participant in the auction cannot affect the probability that the provision point requirement is met by increasing or decreasing their offer or ii) \( v_i = 1 \).


Proof. The expected profit function for the multiunit reverse discriminative auction is given by (2). Let $g(n, p, o_i)$ represent the probability that an offer is one of the $p$ lowest and let $o_{i,TC}^*$ represent the optimal offer, given $v_i$, in a multiunit reverse discriminative auction. Note that $g(n, p, o_i)$ is a decreasing function in $o_i$; the larger $o_i$, the less likely it is one of the $p$ lowest offers. Expected profit for the multiunit reverse discriminative auction is maximized where the first order conditions are met:

$$(o_{i,TC}^* - v_i) = \frac{-g(n, p, o_{i,TC}^*)}{\frac{\partial g(n, p, o_{i,TC}^*)}{\partial o_{i,TC}^*}} \quad (31)$$

Let $w(n, p, B, o_i)$ represent the probability that the provision point requirement will be met and let $o_{i,PP}^*$ represent the optimal offer, given $v_i$, in a provision point reverse auction. Note that $w(n, p, B, o_i)$ is a non-increasing function of $o_i$; as a given offer gets larger, the likelihood that the provision point requirement is met does not increase. Then the first order condition for profit maximization for the PPRA is:

$$(o_{i,PP}^* - v_i) = \frac{-g(n, p, o_{i,PP}^*) \times w(n, p, B, o_{i,PP}^*)}{\frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times w(n, p, B, o_{i,PP}^*) + \frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times g(n, p, o_{i,PP}^*)} \quad (32)$$

Suppose $O_{i,TC}^*(v_i) = O_{i,PP}^*(v_i)$. Then $o_{i,TC}^* = o_{i,PP}^*$ for all $v_i$. Multiplying the top and bottom of the right-hand side of (32) by the reciprocal of its numerator yields

$$(o_{i,PP}^* - v_i) = \frac{-1}{\frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{g(n, p, o_{i,PP}^*)} + \frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{w(n, p, B, o_{i,PP}^*)}} \quad (33)$$

From (31), and given that $o_{i,TC}^* = o_{i,PP}^*$, we have

$$(o_{i,PP}^* - v_i) = \frac{-1}{\left(\frac{-1}{(o_{i,PP}^* - v_i)} + \frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{w(n, p, B, o_{i,PP}^*)}\right)} \quad (34)$$
Multiplying both sides by the denominator of the right-hand side yields

\[
(o_i^*_{i,PP} - v_i) \times \left( \frac{-1}{(o_i^*_{i,PP} - v_i)} + \frac{\partial w(n, p, B, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} \times \frac{1}{w(n, p, B, o_i^*_{i,PP})} \right) = -1 \tag{35}
\]

which simplifies to

\[
0 = (o_i^*_{i,PP} - v_i) \times \left( \frac{\partial w(n, p, B, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} \times \frac{1}{w(n, p, B, o_i^*_{i,PP})} \right) \tag{36}
\]

Equation (36) implies that either \(o_i^*_{i,PP} = v_i\) or \(\frac{\partial w(n, p, B, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} = 0\). Given that \(o_i^*_{i,TC} = v_i\) only when \(v_i = 1\), the two optimal offer functions can be the same only when each participant cannot affect the probability the provision point requirement is met by changing their offer or \(v_i = 1\).

To prove the other direction, suppose that no individual can affect the probability that the provision point requirement is met by changing their offer. Then, by definition, \(\frac{\partial w(n, p, B, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} = 0\) and, using (32)

\[
(o_i^*_{i,PP} - v_i) = \frac{-g(n, p, o_i^*_{i,PP}) \times w(n, p, B, o_i^*_{i,PP})}{\left( \frac{\partial g(n, p, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} \times w(n, p, B, o_i^*_{i,PP}) + 0 \times g(n, p, o_i^*_{i,PP}) \right)} \tag{37}
\]

Simplifying (37) provides

\[
(o_i^*_{i,PP} - v_i) = \frac{-g(n, p, o_i^*_{i,PP})}{\left( \frac{\partial g(n, p, o_i^*_{i,PP})}{\partial o_i^*_{i,PP}} \right)} \tag{38}
\]

which is the first order condition for the multiunit reverse discriminative auction. If instead of assuming that no individual can affect the probability the provision point requirement is met we assume that \(v_i = 1\), the result follows immediately from Axiom 1.
9.4 Proposition 4 Proof

**Proposition 7:** Suppose $O_{i,TC}^*(v_i)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the multiunit reverse discriminative auction with a target of $p < n$ and $O_{i,PP}^*(v_i)$ is a symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of $p < n$ and a budget of $B$. Further suppose that $O_{i,TC}^*(v_i)$ is convex in $v_i$. If a participant in the auction can impact the probability that the provision point requirement is met, then $O_{i,TC}^*(v_i) \geq O_{i,PP}^*(v_i)$ for all $v_i$.

**Proof.** Equations (31) and (32) provide the first order conditions for the optimal offer for an individual competing in a multiunit reverse discriminative auction and a provision point reverse auction, respectively. Note that $g(n, p, o_{i,PP}^*)$ and $w(n, p, B, o_{i,PP}^*)$ are decreasing and non-increasing in $o_i^*$, respectively, so that both $\frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*}$ and $\frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*}$ are less than or equal to zero. We prove by contradiction. Suppose that $o_{i,TC}^* \leq o_{i,PP}^*$. Then, combining (33) and (31), we have:

$$\left(\frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{g(n, p, o_{i,PP}^*)}\right) + \frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{w(n, p, B, o_{i,PP}^*)} > -\frac{g(n, p, o_{i,TC}^*)}{\left(\frac{\partial g(n, p, o_{i,TC}^*)}{\partial o_{i,TC}^*}\right)}$$

Note that both sides of the inequality are positive. Thus, multiplying both sides of the inequality by their reciprocal does not reverse the inequality. The resulting rearrangement is given by

$$\left(\frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{g(n, p, o_{i,PP}^*)}\right) - \frac{1}{g(n, p, o_{i,TC}^*)} \left(\frac{\partial g(n, p, o_{i,TC}^*)}{\partial o_{i,TC}^*}\right) >$$

$$\left(\frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{w(n, p, B, o_{i,PP}^*)}\right) - \frac{1}{w(n, p, B, o_{i,PP}^*)} \left(\frac{\partial w(n, p, B, o_{i,PP}^*)}{\partial o_{i,PP}^*}\right)$$

4 The convexity assumption holds for every set of parameter values I have tested.
Given our assumptions about \( w(n, p, B, o_{i,PP}) \) and \( g(n, p, o_{i,PP}) \), we know that

\[
- \frac{\partial w(n, p, B, o_{i,PP})}{\partial o_{i,PP}^*} \times \frac{1}{w(n, p, B, o_{i,PP}^*)} \geq 0 \tag{41}
\]

and

\[
- \frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{g(n, p, o_{i,PP}^*)} \geq 0 \tag{42}
\]

Returning to (40), if the sum of (41) and (42) is less than the left-hand side of (40), then we know that (42) is also less than the left-hand side of (40).

\[
- \frac{\partial g(n, p, o_{i,PP}^*)}{\partial o_{i,PP}^*} \times \frac{1}{g(n, p, o_{i,PP}^*)} < - \left( \frac{\partial g(n, p, o_{i,TC}^*)}{\partial o_{i,TC}^*} \right) \times \frac{1}{g(n, p, o_{i,TC}^*)} \tag{43}
\]

which further implies that

\[
- \frac{g(n, p, o_{i,PP}^*)}{\partial g(n, p, o_{i,PP}^*)} > - \frac{g(n, p, o_{i,TC}^*)}{\partial g(n, p, o_{i,TC}^*)} \tag{44}
\]

The completion of this proof requires a lemma.

**Lemma 1:** Suppose \( O_{i,TC}^*(v_i) \) is the symmetric Bayesian Nash equilibrium optimal offer function for the multiunit reverse discriminative auction with a target of \( p < n \). Additionally, suppose that \( O_{i,TC}^*(v_i) \) is a convex function. Then the difference between a given optimal offer, \( o_{i,TC}^* \), and its corresponding value, \( v_i \), is a decreasing function in \( v_i \).

**Proof.** Equation (31) provides the first order condition for the optimal offer, given a value \( v_i \), in a multiunit reverse discriminative auction. The left-hand side of (31) provides the difference between an optimal offer and its corresponding value. Taking a derivative with respect to \( v_i \) on both sides yields

\[
\frac{\partial O_{i,TC}^*(v_i)}{\partial v_i} - 1 = \partial \left( \frac{-g(n, p, o_{i,TC}^*)}{\partial g(n, p, o_{i,TC}^*)} \right) / \partial v_i \tag{45}
\]
Proposition 2 states that $O_{i,TC}^*(v_i)$ is an increasing function, and the convexity assumption implies that the second derivative of $O_{i,TC}^*(v_i)$ is positive over the range $[0,1)$ as well. If $\frac{\partial O_{i,TC}^*(v_i)}{\partial v_i}$ was greater than 1 for any $v_i$ in this range, then $\frac{\partial O_{i,TC}^*(v_j)}{\partial v_i}$ would also have to be greater than 1, for any $v_j > v_i$, by convexity. Recall that $O_{i,TC}^*(v_i)$ is bounded below by the 45 degree line and that $O_{i,TC}^*(v_i)$ converges to 1 as $v_i$ converges to 1, by Proposition 1. If the derivative of $O_{i,TC}^*(v_i)$ was ever greater than 1, then $O_{i,TC}^*(v_i)$ would not converge to 1 as $v_i$ converged to 1. Thus, $\frac{\partial O_{i,TC}^*(v_i)}{\partial v_i}$ can never be greater than 1. This fact, along with Equation 45, immediately provides the desired result.

Lemma 1 states that

$$\partial \left( \frac{-g(n, p, o_{i,TC}^*)}{\partial g(n, p, o_{i,TC}^*)} \right) / \partial v_i < 0$$  (46)

The only avenue through which $v_i$ affects $-\frac{g(n, p, o_{i,PP}^*)}{\partial g(n, p, o_{i,PP}^*)}$ is $o_i$. Further, because $o_i$ is an increasing function of $v_i$, we have

$$\partial \left( \frac{-g(n, p, o_{i,TC}^*)}{\partial g(n, p, o_{i,TC}^*)} \right) / \partial o_i < 0$$  (47)

Inequality (47), along with the assumption that $o_{i,PP}^* > o_{i,TC}^*$, implies that (44) is a contradiction.  \hfill \Box